

METODA KONJUGOVANIH GRADIJENATA

p_0, \dots, p_{n-1} određujemo pomoću gradijenta:

$$p_{k+1} = \nabla F(x_{k+1}) - \mu_k \cdot p_k, \quad (\nabla F(x) = Ax - b) \\ = Hx_{k+1} - b - \mu_k \cdot p_k$$

$$p_0 = Hx_0 - b$$

$$p_{k+1} = Hx_{k+1} - b - \mu_k p_k \quad / \cdot (\cdot, Hp_k)$$

$$(p_{k+1}, Hp_k) = (Hx_{k+1} - b, Hp_k) - \mu_k (p_k, Hp_k)$$

$$(p_k, p_k)_{H} = 0$$

$$\Rightarrow \mu_k = \frac{(Hx_{k+1} - b, Hp_k)}{(p_k, Hp_k)} = \frac{(z_{k+1}, Hp_k)}{(p_k, Hp_k)}$$

ALGORITAM METODE:

x_0 - proizvoljno

$$z_0 = Ax_0 - b, \quad p_0 = z_0$$

$$x_{k+1} = x_k - \lambda_k p_k, \quad \lambda_k = \frac{(z_k, p_k)}{(p_k, Ap_k)}$$

$$z_{k+1} = Ax_{k+1} - b$$

$$p_{k+1} = z_{k+1} - \mu_k \cdot p_k, \quad \mu_k = \frac{(z_{k+1}, Ap_k)}{(p_k, Ap_k)}$$

Ⓓ $z_{k+1} \perp p_k$

$$\text{D: } A \cdot / \quad x_{k+1} = x_k - \lambda_k p_k$$

$$Ax_{k+1} = Ax_k - \lambda_k Ap_k \quad / - b$$

$$\underbrace{Ax_{k+1} - b}_{z_{k+1}} = \underbrace{Ax_k - b}_{z_k} - \lambda_k Ap_k$$

$$z_{k+1} = z_k - \lambda_k Ap_k \quad / \cdot (\cdot, p_k)$$

$$(z_{k+1}, p_k) = (z_k, p_k) - \lambda_k (Ap_k, p_k) \\ = (z_k, p_k) - \frac{(z_k, p_k)}{(p_k, Ap_k)} \cdot (Ap_k, p_k) \\ = 0 \quad \blacksquare$$

PRAVOUGAONI SISTEMI LINEARNIH JEDNAČINA

$$Ax = b$$

$$\dim(A) = m \times n$$

$$\dim(x) = n \times 1$$

$$\dim(b) = m \times 1$$

$$\begin{matrix} m \\ \boxed{A} \\ n \end{matrix} \begin{matrix} \boxed{x} \\ n \end{matrix} = \begin{matrix} \boxed{b} \\ m \end{matrix}$$

$m > n$ PREODREĐEN
 $m < n$ NEODREĐEN
 ...
 ...
 ...

$m > n$: neke jednačine nisu zadovoljene
 želimo da min grešku $z = Ax - b$

$$\begin{aligned}
 \text{Greška: } F(x) &= \|z\|_2^2 = \|Ax - b\|_2^2 = (Ax - b, Ax - b) \\
 &= (Ax, Ax) - (b, Ax) - (Ax, b) + (b, b) \\
 &= (Ax, Ax) - (A^T b, x) - (x, A^T b) + (b, b) \\
 &= (A^T A x, x) - 2(A^T b, x) + \underbrace{(b, b)}_c \\
 &= \frac{1}{2} (2A^T A x, x) - 2(A^T b, x) + c \rightarrow \min
 \end{aligned}$$

Realne vrednosti

Funkcional $\frac{1}{2}(Ax, x) - (b, x)$ dostiže min za ono x^* koje je $Ax^* = b$

$$\Rightarrow 2A^T Ax = 2A^T b$$

$$\Rightarrow \boxed{A^T A x = A^T b}$$

$A^T A$ je Hermitova pozitivno def. matrica ako je A punog ranga ($\min\{n, m\}$)

$$\begin{aligned}
 \mathbb{C}: F(x) &= \overbrace{(Ax - b, Ax - b)}^{e \in \mathbb{R}} \\
 &= \overbrace{(Ax, Ax)}^{e \in \mathbb{R}} - \overbrace{(Ax - b)}^{e \in \mathbb{R}} - \overbrace{(b, Ax)}^{e \in \mathbb{R}} + \overbrace{(b, b)}^{e \in \mathbb{R}} \\
 &= (Ax, Ax) - (b, Ax) - (b, Ax) + (b, b) \\
 &= (A^* A x, x) - 2 \operatorname{Re}(b, Ax) + \underbrace{(b, b)}_c \rightarrow \min
 \end{aligned}$$

$$\operatorname{Re}((A^* A x, x) - 2(A^* b, x)) \rightarrow \min$$

$$\operatorname{Re}(A^* A x - 2A^* b, x) \rightarrow \min$$